I. B. Pestov*
Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research
Dubna, 141980, Russia
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In this work, the experiment is discussed on the verification of the principle of universality of gravitational interactions and some related problems of gravity theory and physics of elementary particles. The meaning of this proposal lies in the fact that the self-consistency of General Relativity, as it turns out, presuppose the existence of the nongravitating form of energy. Theory predicts that electrons are particles that transfer the nongravitating form of energy.

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I. INTRODUCTION

The modern stage of development of gravity theory is characterized not only by the search for new effects and new experiments but also by a deeper analysis of fundamentals of the theory and conceptual problems including an important problem of the energy of gravitational field [1,2,3]. Difficulties caused by the nontensor character of quantities describing the energy of gravitational field turn out to be so serious that they are considered as manifestation of specific properties of the gravitational field: universality, nonscreening nature, nonlocalizability. A detailed analysis shows that none of specific properties of gravitational field can explain the so-called nonlocalizability of that field. Not only the energy but also all the results of theory, except for the Lagrange function and gravitational field equations, appear to be noncovariant. So, in general relativity, a nonstandard situation occurred: in the theory whose principles are mathematically formulated rigorously, important physical consequences are in contradiction with the initial statements.

So, when general relativity is formulated, a general logical requirement admissibility of arbitrary systems of coordinates is postulated, however, it turns out that in the constructed theory, the dynamic characteristics of the gravitational field (except for the Einstein equations), the density of energy and momentum, are described by nontensor quantities. As a result, it is impossible to uniquely describe the distribution of energy-momentum of any physical system in the gravitational field. Therefore, there occurs the notion of nonlocalizability of the gravitational field. The energy of this field is not localizable, i.e. there is no uniquely defined energy density.

II. NONLOCALIZABILITY

The nature of this phenomenon is as follows. While the electromagnetic field is described both by a vector potential and a metric, the Einstein law of gravity [4] does not contain anything except gravitational potentials. For the electromagnetic field, the physical quantity is the class of equivalence of vector potentials determined by one arbitrary function. A representative of each class of equivalence is chosen by imposing the Lorentz condition that is generally covariant, i.e. independent of a particular system of coordinates, since the theory contains the so-called background object, the Minkowski metric. At the same time, the physical quantity for gravitational field is the class of equivalence of gravitational potentials defined by four arbitrary functions. Only one quantity, the Einstein-Hilbert action, is independent of the choice of these functions. Extending analogy, we note that different representatives of the class of equivalence in the Maxwell theory correspond not only to the same action but also to the so-called tensor of electromagnetic field. Therefore, the same Lorentz force and energy density correspond to different representatives of the equivalence class. In the Einstein theory, different representatives of the equivalence class correspond to the same gravitational field that is differently oriented in space-time with respect to the same observer. Different representatives correspond to different orientations. Ambiguity in the choice of orientation is determined by four arbitrary functions of coordinates. Since the theory does not contain any objects besides gravitational potentials, the representative from

*Electronic address: pestov@thsun1.jinr.ru

each equivalence class can be chosen in a general covariant manner only through introducing, into the theory, the nondynamic so-called background object, background metric [5].

The choice of a representative from each equivalence class is achieved by imposing four general covariant conditions on covariant derivatives of gravitational potentials with respect to the background connection. Nonlocalizability of the gravitational field is then defined by the freedom in choice of the background metric or background connection. Thus, the problem of energy of the gravitational field is reduced to the problem of physical meaning of the background connection that becomes of principal importance. If the gravitational field and background connection are given on the same manifold, gravitating particles are moving along geodesics defined by gravitational potentials. Then, a natural question arises concerning the nature of particles moving along geodesics given by the background connection. Existence of particles of that sort is an evident necessity without which it is a difficult problem to discuss the physical meaning of the background connection. The latter problem can be avoided by assuming that the particles are moving along geodesics of the background connection when there is no gravitational field. Then it follows that the background connection has the meaning only in the absence of gravitational field.

So, based on purely logical requirements following ¿from known facts, we conclude that there exists nongravitating form of the energy that is directly related to the physical meaning of gravitational potentials in the framework of general covariance principle. As it has already been underlined, the latter is a purely logical requirement inherent in any physical theory, including gravity theory. Hence it becomes necessary to experimental verify the universality principle of gravitational interactions. "The validity of this principle in the microscopic domain is not as evident as the validity of the coincidence axiom. We know of many rules which apply with great rigor to electromagnetic and other types of interaction and it is conceivable that the special role of the gravitational interaction may dissolve in higher harmony. For this reason, I shall pay prime attention to Einstein's first observation, that only coincidences have a direct physical meaning, values of coordinates do not." This citation is taken from paper by Wigner [6] in order to emphasize that it is important to test the universality principle of gravitational interactions not only for the problem of self-consistency of general relativity but also for explanation of the role of gravitational forces in the physics of microworld.

III. ON GRAVITY LAW

We will mathematically formulate main distinctive features of general relativity on the basis of the Einstein gravity law [4]. This law, as viewed in the framework of mathematical analysis, is the following system of nonlinear second-order partial differential equations

$$R_{ij} = 0 \tag{3.1}$$

for ten functions $g_{ij}(x)$ of 4 independent variables x^0, x^1, x^2, x^3. This system of equations possesses the following remarkable property. Let the functions $g_{ij}(x)$ have a common domain of definition D and are a solution to system (1). Now, consider functions $\varphi^i(x), \quad i = 0, 1, 2, 3$, such that their domain of definition and range of their values contain D, and the Jacobian $J = |\partial\varphi^i/\partial x^j|$ differs from zero in D. As it is known, in this case in the domain D there exist functions $f^i(x)$, such that

$$\varphi^i(f(x)) = x^i, \quad f^i(\varphi(x)) = x^i.$$

Next, let us constitute the functions

$$\tilde{g}_{ij}(x) = g_{kl}(f(x)) f^k_i(x) f^l_j(x), \tag{3.2}$$

where $f^k_i(x) = \partial_i f^k(x)$. Substitution of (2) into (1) shows that the functions $\tilde{g}_{ij}(x)$ are a new solution to equations (1) provided that the functions $g_{ij}(x)$ in (2) are a solution to those equations. This analytic aspect of the gravity law is a crucial point. We will now consider its manifestation in various problems.

First, we shall examine the very important Cauchy problem for eqs. (1). The metrics $g_{ij}(x)$ and $\tilde{g}_{ij}(x)$, as mentioned above, describe the same physical situation. It is associated with the whole class of equivalence of solutions to eqs. (1) determined by 4 functions. To choose a certain element from every class of equivalence, it is convenient to introduce a global "background" metric and to impose 4 conditions on covariant derivatives of the physical metric with respect to the Levi-Civita background metric $\hat{g}_{ij}$ [8]

$$\hat{\nabla}_i(\sqrt{-g} g^{ij}) = 0,$$

which remove arbitrariness defined by eqs.(2). The result is reduced equations of the hyperbolic type for the metric g_{ij} with respect to the global background metric $\hat{g}_{ij}$. The background metric is quite necessary for writing "gauge conditions" in a general-covariant form independent of the choice of a coordinate system. So, to derive a definite solution to the Einstein equations (1), one should eliminate arbitrariness given by analytic relations (2), which is achieved as indicated above. Consequently, to obtain physical solutions to equations of the gravitational field, they should be treated as a nonlinear system of second-order equations on the manifold M with metric $\hat{g}_{ij}$ given on the manifold globally [6]. Note that the notion of a continuous 4-dimensional manifold provided an effective remedy used in considerations in modern physics. The very structure of manifold with its topology remains arbitrary and should be determined by considerations, generally speaking, outside of the scope of general relativity. Thus, the Cauchy problem in general relativity can be brought into the general-covariant form necessary for any physical theory only when the background connection is introduced.

For further clearer illustration, let us trace analogy with the theory of gauge fields. According to Einstein, the gravitational field is put into correspondence with a symmetric tensor field of the second rank g_{ij}, satisfying the nonlinear equations (1). The electrotonic state of electromagnetic field introduced by Faraday is described by the vector potential (1-form) $A = A_i dx^i$. The gauge transformations

$$A \rightarrow \bar{A} = A + d\varphi$$

correspond to transformations (2). Principal difference of the gravitational field from the electromagnetic field is that one cannot construct quantities invariant under transformations (2) from the Einstein gravitational potentials, whereas from components of the vector potential, the gauge-invariant tensor of electromagnetic field (2-form) $F = dA$ can be constructed. In particular, with the help of (2) it is not difficult to verify that the tensor of Riemannian curvature that is in a sense analogous to the tensor of electromagnetic field is not invariant under the transformations of gravitational potentials (2). The case of non-Abelian gauge fields, in the sense of existence of gauge-invariant quantities, is closer to general relativity as compared to the Maxwell theory. The reason is that the strength tensor of a non-Abelian gauge field is not a gauge-invariant quantity [10]; only the energy-momentum tensor and Lagrange function remain gauge-invariant quantities. This can be observed as follows: with a non-Abelian gauge group, one cannot connect conserved gauge-invariant "charges" analogous to the electron charge. This is a principal difference of the Yang-Mills field theory from electrodynamics, and just in this it is closer to general relativity, where also one cannot introduce an invariant gravitational "charge". It is well-known which difficulties are connected with the extension of gauge symmetry and how they were overcome. A rather long way was required to construct physically acceptable models with non-Abelian gauge fields. Spontaneous breaking of symmetry as the mechanism that allows one to provide quanta of those fields with mass and localization of interactions of the indicated class are merely the most typical manifestations caused by the change of the gauge group of electrodynamics to a wider gauge group of transformations. In gravity theory, only the Einstein-Hilbert action and equations of the gravitational field deduced from it are invariant under transformations (2). Related difficulties are not yet overcome. Like in the theory of non-Abelian gauge fields, in gravity theory, one cannot introduce a gravitational charge invariant under transformations (2). Which are consequences and how they show themselves physically, is still an open problem. The first important step along this way could be the problem of existence of a nongravitating form of energy, because having solved this problem, one could raise the problem of physical meaning of the background connection and analyze the corresponding consequences. In this connection, consider evidences for existence of a nongravitating form of energy beyond the scope of general relativity.

IV. NONGRAVITATING FORM OF ENERGY

As it is shown in [7], the existence of nongravitating form of energy is closely related to gauge symmetry inherent in general relativity. The idea of symmetry like that is very simple and consists in that coordinates on the total linear group can be put in correspondence with tensor fields of type (1,1) on a manifold rather than a set of scalar fields , it is accepted in gauge abstract theory. Hence, it follows that coordinates of all subgroups of the total linear group can also be put in correspondence with tensor fields on a manifold.This actually covers all physically interesting groups of gauge symmetry. The gauge symmetry under consideration is a realization of the abstract theory of gauge fields in the framework of modern differential geometry. Its typical feature is that it does not supposes distinction between space-time and gauge, or so-called isotopic, space. At the same time contemporary gauge models presuppose an exact local distinction between space-time and gauge space. Just in this aspect, the gauge symmetry under consideration opens essentially new possibilities. So, the total linear group and its subgroups admit a simple realization in terms of geometrically well defined tensor fields.

The total linear group, like the group of gauge symmetry, is invariant under transformations (2). This signifies that transformations (2) do not break equivalence given by that gauge group. However, any reduction of the considered gauge group to its subgroups results in that all these subgroups are not invariant under transformations of the symmetry group of gravitational interactions defined locally and usually called the group of diffeomorphisms. Thus, if at least one subgroup of the gauge group is physically realized, the form of energy connected with this system will be nongravitating. In the papers mentioned above it has been shown that the Dirac theory cannot be deduced without reduction of the given gauge group. This result is itself rather easily apprehended within the Dirac theory on the basis of the well-known fact that spinors are not a basis of a representation of the total linear group considered here as a group of transformations of coordinates. For that reason, difficulties have arisen of fundamental character when introducing spinors into general relativity. Strange though it may seem, the way out of that situation was looked for not through analyzing and verification of the universality principle of gravitational interactions as applied to electrons but through introducing an orthogonal basis into gravity theory. The difficulties with geometrization of the Dirac theory were resolver in its favor.

Accurate formulation of the Dirac theory, even in the Minkowski space-time, requires introduction of the so-called tetrad. As a result, instead of the analysis of the Dirac theory, there appear various tetrad theories of gravity and, which is surprising, the introduction of basis was apprehended as a peculiar revelation not only in physics, but in geometry, as well. However, a pure logic and evident requirement applicable to any physical theory is the principle according to which physical laws should be formulated in the form independent of both the choice of a coordinate system and a basis in vector spaces associated with a given theory. The notion of basis should be absent in the formulation of physical laws. For the first time, this principle was formulated by Dirac who gave a consistent construction of quantum mechanics on its basis [8]. However, it turns out that the Dirac electron theory does not the principle he formulated. It seems that existence of the Dirac theory very clearly points out that this theory is a reduction of a more general theory and the simplest way to perform that reduction is to employ an orthogonal basis. This program was realized in the papers mentioned above. The relevant conclusion consists in that electrons carry the nongravitating form of energy. So, the nature unambiguously points to self-consistency of general relativity and to particles that transfer the nongravitating form of energy, electrons.

V. ON GRAVITATIONAL INTERACTIONS OF ELECTRONS

We will not present detailed computations, rather we dwell upon main results following from [7]. It has been shown there that the theory of fermion fields cannot in principle be formulated without reduction of the total linear group as a group of gauge symmetry. As a result, it turned out to be impossible to construct the theory invariant under both gauge transformations and transformations of the group of diffeomorphisms. This result could be easily understood without formulae, as well. Reduction of the gauge group is accompanied by imposing constraints formulated in terms of the metric tensor. Consequently, the reduced gauge group will be invariant under transformations of the group of diffeomorphisms if the metric tensor does not change under those transformations. According to (2), the condition for the metric being conserved under diffeomorphisms is expressed by the system of partial differential equations for functions...

$$\tilde{g}_{ij}(x) = g_{kl}(f(x)) f^k_i(x) f^l_j(x) = g_{ij}(x). \tag{5.1}$$

Equations (3) may not have solutions at all. If $g_{ij}(x)$ in (3) is the Minkowski metric, the group of space-time symmetry that keeps the reduced gauge group to be invariant will be the Poincare group which is in the given case a general solution to equations (3).

When the problem of gauge-invariant determination of the energy density was studied, it was proved that there exists canonical gauge-invariant tensor of the energy-momentum; the latter differs from the metric tensor of energy-momentum which is not gauge-invariant since reduction of the gauge group breaks gauge invariance of the procedure of deriving the metric tensor of energy momentum. As the right-hand side of the Einstein equations contains the metric tensor of energy-momentum, it follows that gravitational interactions cannot be introduced in a gauge-invariant way. As noted above, the Dirac theory is realized in the reduction of the total linear group as the group of gauge symmetry. However, the reduction of this gauge group automatically leads to the reduction of the group of diffeomorphisms which is a symmetry group of gravitational interactions.

Now, we will formulate conditions to hold for any gravitational-interacting field. Let a physical object be described by an appropriate field, and equations of the field be derived from the Lagrangian. Then the field is gravitational-interacting provided the following condition are fulfilled. The considered field, like the gravitational field, makes a basis of the faithful representation of the diffeomorphism group. Varying the action of the field with respect to the metric, we obtain the metric tensor of energy- momentum. This symmetric tensor will obey the well-known local conservation law on solutions to the field equations. If the theory has any gauge symmetry, the energy-momentum tensor should be gauge-invariant. A relevant necessary condition is invariance of the gauge group with respect to transformations of the group of diffeomorphisms, the symmetry group of gravitational interactions. If the latter condition is not satisfied, there are two possibilities. The first consists in that gauge symmetry is eliminated from consideration, and the theory is constructed which is invariant only with respect to transformations of the group of diffeomorphisms. The second possibility is to construct the gauge-invariant theory which cannot include gravitational interactions. This means that there exists the nongravitating form of energy. As it has turned out, the Dirac theory does not obey the formulated conditions.

The Dirac theory unambiguously points to the existence of nongravitating form of energy and to a concrete object , the carrier of that form, the electron. Reduction of the gauge group resulting in the Dirac wave function, (for details see papers cited above) reduces the group of space-time symmetry step by step, so that the latter is to be properly restored to ensure the Poincare invariance. This is achieved with the help of global gauge invariance, and therefore, the electron represents the nongravitating form of energy.

The simplest test for establishing whether the electron is that form is to measure its weight. Experiments of that kind were performed in 1967 at Stanford University by the Fairbank group [9]. They demonstrated a zero result. Here we will cite the abstract of paper [9].

"A free-fall techniques has been used to measure the net vertical component of force on electrons in vacuum enclosed by a copper tube. This force was shown to be less than $0.09mg$, where m is the inertial mass of the electron and g is $980cm/sec^2$. This supports the contention that gravity induces an electric field outside a metal surface, of magnitude and direction such that the gravitational force on electrons is cancelled".

Thus, the explanation of experimenters of that "zero" is that gravity induces, in an experimental setup, an electric field which just compensates the force of gravitational attraction. It follows then that positrons under the same conditions should fall with acceleration $2g$. Fairbank planned to carry out experiments with positrons, however, this plan met with difficulties. Later, when up-to-data-technologies open new possibilities, preparation of this experiment was taken up again. Unfortunately, the experiment with positrons remained at the level of preparation since after 1989 it was closed because of decease of the project's leader. Details of the Fairbank experiment and subsequent discussions can be found in review papers [10] and [11].

VI. PROPOSAL OF EXPERIMENT

What is suggested on the basis of our consideration? To solve the problems of fundamental importance both for gravity theory and elementary particle physics, it is necessary to perform the Fairbank experiments both with electrons and positrons, i.e. to measure the acceleration of electrons and positrons in the gravitational field of the Earth. If the result is zero, in accordance with theoretical predictions, it will first of all follow that general relativity is , like the theory of electromagnetic fields, self-consistent. An urgent necessity will be the problem of physically substantiated choice of the background metric and proper violation of the symmetry of gravitational interactions, of the physical meaning of that manifold, with which the background metric is connected. Consideration of gravitational interactions on the background of the Minkowski metric is quite natural. However, other possibilities should not be neglected a priori, especially, in connection with the problem of compatibility of gravitational field equations and subsidiary (auxiliary?) conditions imposed on gravitational potentials [5]. It may happen that gravitational interactions, like weak interactions, are localized, which means that gravitons transfer the mass. In the physics of elementary particles, it will be necessary to consider the problem of gravitational interactions of the emitting matter as a collective effect.

In conclusion, we note that expenditures on the experiment proposed could by no means be compared with its fundamental importance for physics, we tried here to demonstrate from various viewpoints.

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